HSC Research Report

HSC/97/01

Evolution in a changing environment

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Abstract: We propose a simple model, based on Monte Carlo simulations, for studying the effects of changes in the environment on the adaptation and extinction of evolving species. We show that the geological data of climatic changes are well described by Lévy-stable distributions. This leads, in our model, to a fairly good reproduction of the known data on species extinctions. We have also found that the dependence of the probability that a given number of species becomes extinct in one time step, on the number of extinct species shows a cross-over from an exponential to a power-like character.

PACS: 0.2.70.Lq, 87.10.+e
Keywords:

1 Introduction

Many interesting results have been recently obtained by physicists studying systems modeling biological evolution. In most of them, following the now classical idea of Bak and Sneppen [1], an ecosystem is portrayed by a set of species whose characteristics, often called fitness, are randomly changed [2, 3, 4] and the system exhibits self-organized criticality (SOC). In another approach one is studying the influence of changing phenotypes on the evolution of the species [5, 6, 7]. One of the main ideas following also from SOC is that evolution shows punctuated equilibrium rather than gradual development [8]. The problem has a century old history, going back to the works of Lyell and Darwin (see [8] for a more general discussion). Although it is quite obvious that biological evolution is very strongly influenced by e.g. climatic changes, little attention has been paid by physicists to the problem [4]. In this letter we propose a simple model of an evolving system of species under the influence of a changing environment. We want to find out, first of all, what is the best way to describe climatic changes, then how they influence the evolution of species. In particular how the adaptation will respond to climatic changes and above all whether the extinction of species in our model will bear any resemblance to the recorded biological data [9, 10]. Temporal variance of the extinction of species has been also reported by Sneppen et al. [15], however, without relating it to the changes of the environment. We shall use the Monte Carlo simulation technique.

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2 The model

We consider a system of N species, each characterized by a number $a_i \in (0,1)$, $i = 1,\ldots,N$, which gives the adaptation of the i-th species to the environment. The idea of adaptation as a similarity of the phenotype of a population to the 'ideal' phenotype of the environment was introduced in [7]. Changes in the environment impose modifications of the species, hence variations of their adaptations. According to palleoclimatologists [11] '...brief periods of rapid step-like, climatic changes appear to separate seemingly stable interludes ...' (Fig.1).

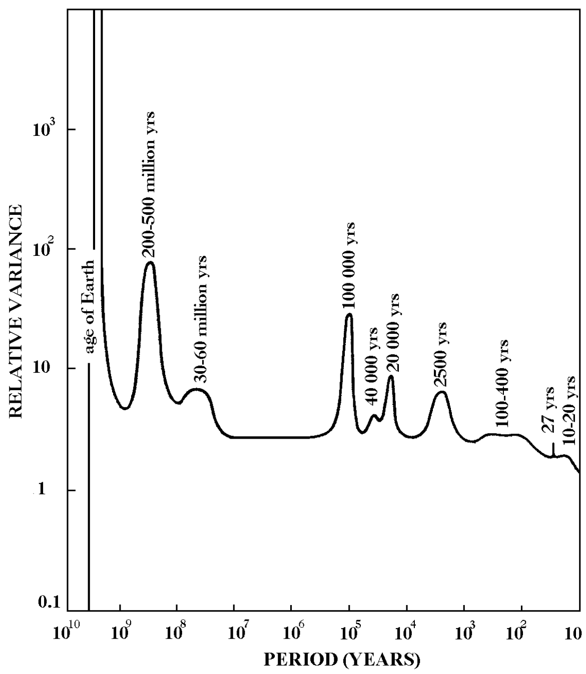

Figure 1: Relative climate variation in the last 10,000 million years. Source: [11].

Such a behavior can be observed in many natural systems. In 1919 a Danish astronomer J. Holtsmark [12] noticed that gravitational fields of stars are distributed in such a way that the 3D Fourier transform of their density, $p(x)$, has a simple form

$$\int_{R^3} \exp\{itx\}p(x)dx = \exp(-\lambda|t|^{3/2}). \tag{1}$$

We now know that it corresponds to the so called symmetric Lévy-stable distribution with $\alpha = 3/2$. In 1960 Mandelbrot [13] suggested the use of such distributions for modeling complex systems with large events, see also [8].

The symmetric Lévy-stable (also called α-stable) distribution $S_{\alpha,c}$ is characterized by two parameters [14]. The *scale parameter* $c > 0$ compresses or extends the distribution about 0. The *characteristic exponent* α lies in the range $(0, 2]$ and determines the rate at which the tails of the distribution taper off. When $\alpha = 2$, the well known normal (Gaussian) distribution results, with mean 0 and standard deviation $\sqrt{2}c$. When $\alpha < 2$, the distribution shows 'longer' ('heavier') tails, allowing for occurrence of unlikely events. In physical literature such tails are often described as Lévy-Pareto, since they decrease at the same rate as those of the Pareto distribution. When $\alpha = 1$, the well known Cauchy distribution is obtained with location 0 and scale c.

In our model, changes in the adaptation of the species follow from a simple rule: at each time step t we change a_i by a random value r_i chosen from the normal distribution with mean 0 and standard deviation $\sigma(t)$

$$a_i(t+1) = a_i(t) + r_i. \tag{2}$$

If $a_i(t+1) < 0$ or $a_i(t+1) > 1$ the i-th species is eliminated, because it is either too badly or too well adapted. In place of the eliminated species a new one is selected with adaptation being a random number between 0 and 1. The changes of the environment are not uniform but show large peaks, which in the case of global climate changes may correspond to catastrophic events like hitting of the Earth by a large meteorite, a big scale volcanic eruption or rapid, on the geological time scale, climatic changes [11]. Such events have global effects and may produce extinction of wide-spread species. Therefore, it seems natural to let $\sigma(t)$ vary over time according to a distribution allowing for occurrence of unlikely events, yet at the same time giving a good fit to real life data.

At each time step t we let $\sigma(t)$ be the absolute value of a random variable chosen from the symmetric Lévy-stable distribution $S_{\alpha,c}$. For the 1-stable (Cauchy) distribution we can easily derive the formula for the probability density function $f_r(x)$ of r_i

$$f_r(x) = \frac{2}{\pi}\int_0^\infty \frac{1}{y\sqrt{2\pi}} \exp\left(-\frac{x^2}{2y^2}\right) \frac{c}{c^2+y^2} dy, \tag{3}$$

and for the cumulative distribution function $F_r(x)$ of r_i

$$F_r(x) = \frac{2}{\pi}\int_0^\infty \Phi\left(\frac{x}{y}\right) \frac{c}{c^2+y^2} dy. \tag{4}$$

Here $\Phi(x)$ stands for the standard normal (mean 0 and standard deviation 1) cumulative distribution function.

On the other hand, choosing r_i from a symmetric Lévy-stable distribution with $\alpha < 2$ (and not from a normal distribution) does not influence the results in any significant way. The same is true if we take a uniform distribution.

A single Monte Carlo step in our algorithm consists of the following:

1. let $\sigma(t)$ be the absolute value of a random variable chosen from the symmetric Lévy-stable distribution,
2. for each species $i = (1, \ldots, N)$ pick a random number r_i normally distributed with mean 0 and standard deviation $\sigma(t)$,
3. calculate the new adaptation of the i-th species

   $$a_i(t+1) = a_i(t) + r_i$$

   . If $a_i(t+1) < 0$ or $a_i(t+1) > 1$, the species is eliminated and a new one appears with randomly chosen adaptation a_i.

In the simulations we have generally dealt with $N = 10,000$ species and the averaging has been done over 250,000 independent runs.

3 Results

It is natural to believe that equilibrium in nature exists. In our model, this phenomenon is expressed in the nearly constant value of the mean adaptation $A(t)$ defined as

$$A(t) = \frac{1}{N}\sum_{i=1}^{N} a_i(t) - A(0), \tag{5}$$

see Figs.2-3(b). Deviations from the mean exist. However, the larger population we consider the smaller deviations will be observed

$$\lim_{N\to\infty}\sum_{i=1}^{N} r_i = 0.$$

Rapid, on the geological time scale, climatic changes (Figs.2-3(a)) may produce extinction of wide-spread species. However, new species appear in place of the old ones. The percentage of extinct species $D(t)$ at time t is plotted in Figs.2-3(c)

$$D(t) = \frac{1}{N}\sum_{i=1}^{N} d_i(t), \tag{6}$$

where $d_i(t)$ is zero if at time t the i-th species is still living and it is equal to one if the species died out at time t.

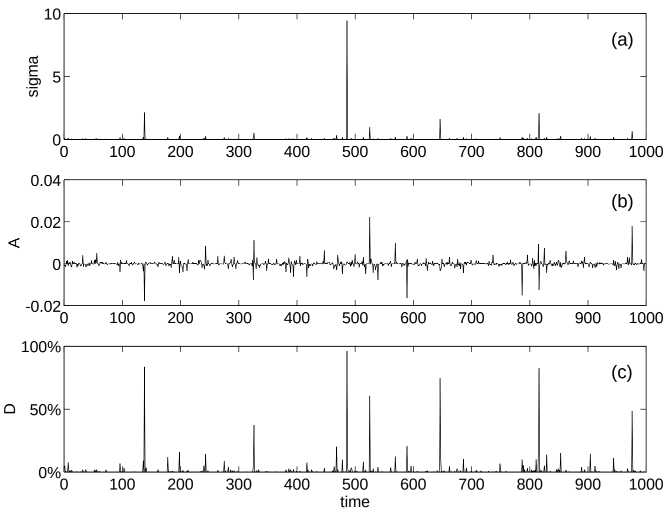

Figure 2: (a) Climate changes ($\sigma(t)$), (b) mean adaptation, and (c) percentage of extinct species for the 1-stable (Cauchy) distribution. Decreasing α results in larger, on the geological time scale, climatic changes separated by longer 'seemingly stable interludes'.

Fig.2 shows that in general a maximum in $\sigma(t)$ is followed by a peak in either adaptation or extinction, or both. However, the relation is not obvious – there is no evident correlation between the height of the $\sigma(t)$ maximum and the peaks in $A(t)$ or $D(t)$.

This is certainly true for the extinction diagram (Fig.2(c)), where 'mass extinctions' may be caused by relatively small changes of the climate. Although, all big-scale climate variations

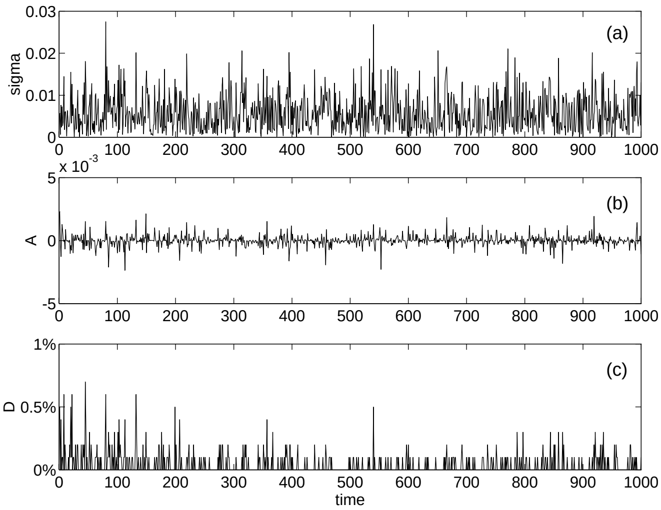

Figure 3: Same as in Fig.2, but for the 2-stable (normal) distribution. It is clear that the normal distribution shows no 'rapid changes followed by stable interludes' character and neither any resemblance to the climatic changes.

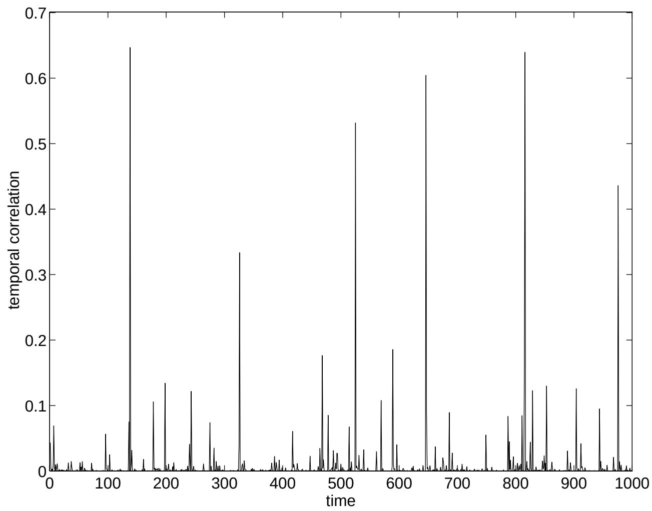

Figure 4: Temporal correlation $I(t)$ between $\sigma(t)$ and $D(t)$ for Fig.2.

lead to appreciable extinctions of the species. To have a more quantitative description of the relations between environmental changes and extinction, we introduce a function $I(t)$ measuring the temporal correlations between two, defined as

$$I(t) = \frac{D(t)}{D_{max}} - \frac{\sigma(t)}{\sigma_{max}}. \tag{7}$$

D_{max} and σ_{max} denote the maximal values of $D(t)$ and $\sigma(t)$, respectively, recorded during the simulation. $I = 0$, as always, means the total correlation (e.g. no change in the climate brings no extinctions), $I < 0$ signifies that a large climate change produces only small-scale extinctions, whereas $I > 0$ means the opposite, mass-extinctions provoked by only minor climate changes.

The rather complicated behavior found through simulations may be also reproduced, to some extent, in an analytical, mean-field like, approach. For a large number of species we may safely assume that, at each time t, the average adaptation $\frac{1}{N}\sum_{i=1}^{N} a_i(t)$ oscillates around 0.5. We may further assume that the probability of extinction of the i-th species (i.e. $P(d_i = 1)$) is equal to the probability $P(|r| > 0.5)$

$$P(d_i = 1) = 1 - \int_{-0.5}^{0.5} \frac{1}{\sigma\sqrt{2\pi}} \exp\left(\frac{r^2}{2\sigma^2}\right) dr. \tag{8}$$

From this relation we may obtain the dependence quite similar to the pattern in Figs.2-3(c).

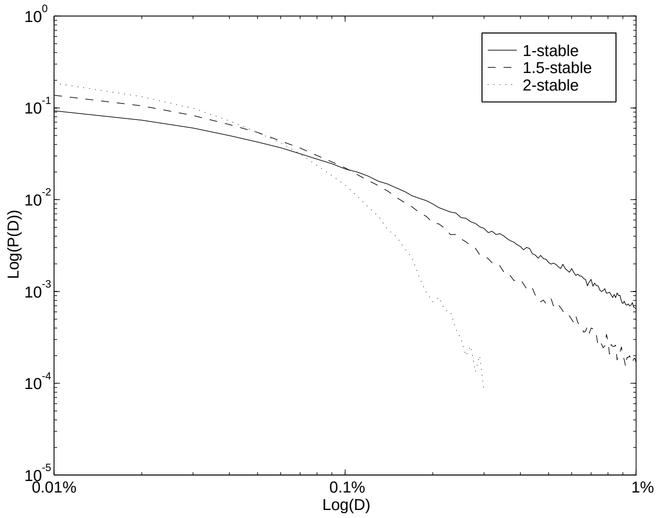

Figure 5: Probability that $D\%$ of the species will be eliminated in one time step.

The dependence of the probability $P(D)$ (probability that $D\%$ of the species becomes extinct in one time step) on percent of extinct species D shows a rather complex character (Fig.5). For $D < \tilde{D} \approx 0.1\%$ the dependence can be, to a good degree, approximated by an exponential

$$P(D) \sim C_1(\alpha) e^{-\delta(\alpha) D} \tag{9}$$

whereas for $D > \tilde{D}$ it has a power-like character

$$P(D) \sim C_2(\alpha) D^{-\gamma(\alpha)}. \tag{10}$$

The change in the functional fit to $P(D)$, seems to indicate a difference in the respective 'mechanisms'. When the changes are small, the system remains in equilibrium and as such is described by a normal distribution (exponential fit). When larger changes occur, the system may become unstable and hence normal distribution cannot be applied. Segments of $D < \tilde{D}$ may correspond to those 'stable interludes' in climate changes and smoothly proceeding evolution. The power-law behavior is analogous to the Gutenberg-Richter law describing earthquakes [16]. Generally (like in the Gutenberg-Richter law) one investigates only the tails of the distribution, hence the exponential part is never mentioned.

Bak and Paczuski [8] estimated the value of the exponent γ (from the data given by Raup [9]) as $1 \leq \gamma \leq 2$. This according to Fig.6 corresponds to values of α between 0.5 and 1.5, hence rather long intervals of small changes, interrupted by sudden bursts (see Figs.2-3).

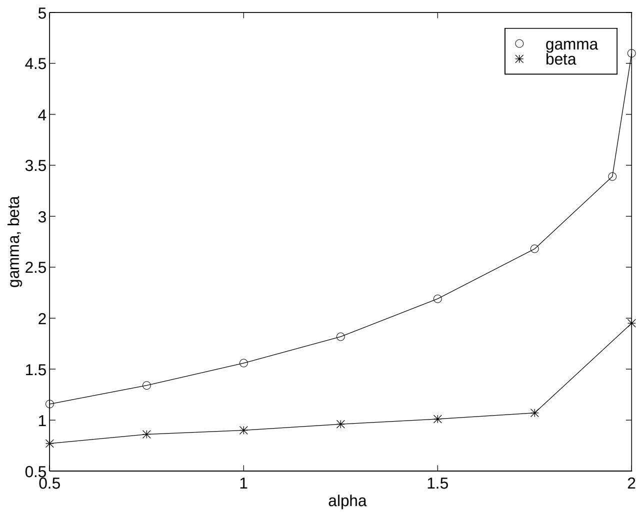

Figure 6: Dependence of indices β and γ on the α parameter.

Analogous calculations as for the probability $P(D)$ of extinct species, can be made for the probability $P(n)$ of species which changed their adaptation. This dependence has a power-like character

$$P(n) \sim n^{-\beta}, \tag{11}$$

see Fig.6.

4 Conclusions

We have proposed a simple, simulation based model for studying the influence of the changes in the environment (climate) on the adaptation and extinction of evolving species. The number of (non-interacting) species is constant. The results were found to be independent of that number.

We have found that Lévy-stable distributions, characterized by $\alpha < 2$, describe correctly the non-uniform character of climate changes. In doing so we are taking side of the advocates of punctuated equilibrium evolution as opposed to the more traditional concept of gradual evolution (see e.g. [8]). The climate variations are related in a simple way to the changes of the adaptation of the species. We have shown that in our model the distribution of the extinct species agrees well with the known biological data, showing a power law behavior. The value of the index depends on the character of the climatic changes in an almost linear way, except if the climate changes distribution is close to a normal one.

We have also shown that, at least in our model, the relation between environment changes and extinction pattern is not a straightforward one. It happens that mass extinctions are provoked by apparently small disturbances in the climate. However, always the biggest extinction can be associated with the largest climate change. We hope that despite leaving aside many important factors, the fact that a so simple model produces results which agree quite well with the known data, may validate our assumptions and make the model useful.

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